

# Low-lying states in $^{195}\text{Pt}$ described by the particle-rotor model

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Received: 13 December 2005 / Revised version: 6 February 2006 /

Published online: 21 February 2006 – © Società Italiana di Fisica / Springer-Verlag 2006

Communicated by R. Krücken

**Abstract.** Particle plus triaxial rotor model calculations have been performed for the negative-parity states in  $^{195}\text{Pt}$ . An overall reasonable description of the level energies below 700 keV and of the corresponding  $B(E2)$  and  $B(M1)$  transition strengths was obtained. The doublet structure of the levels may be associated with the closeness in energy of two active negative-parity neutron quasi-particles which are positioned near to the Fermi surface for  $\gamma \approx 30^\circ$ . The detailed comparison with the data reveals that further theoretical efforts, especially to account for the properties of the core and the particle-core coupling, are required for a more successful description of  $^{195}\text{Pt}$  in a particle plus rotor model.

**PACS.** 21.60.-n Nuclear structure models and methods – 23.20.Lv gamma transitions and level energies – 27.80.+w  $190 \leq A \leq 219$

## 1 Introduction

The nucleus  $^{195}\text{Pt}$  has attracted much attention for decades. Many theoretical approaches have been tried in order to explain the complex structure of this even-odd nuclide. Recently [1,2], an extended set of experimental information was again interpreted as a realization of the  $U(6/12)$  supersymmetry (SUSY) for the case of  $^{194}\text{Pt}$  and  $^{195}\text{Pt}$ . The grouping in doublets of the negative-parity levels below 700 keV excitation energy and the description of the measured spectroscopic strengths strongly supports these conclusions. Since the introduction by Iachello of the ideas of boson-fermion supersymmetry [3] in nuclear physics [4,5], group theoretical approaches have dominated the theoretical study of  $^{195}\text{Pt}$  (cf. references in sect. 3). Very recently, the structure of  $^{195}\text{Pt}$  was also discussed in terms of pseudospin symmetry and supersymmetry [6]. Anterior to these efforts, other models have been tried based on the coupling of the odd neutron to a core characterized by a static or dynamic quadrupole deformation [7,8]. Their success has been limited in general, on the one hand, by numerical problems in the calculations, due to the impossibility of using a large enough set of basis states at the time, on the other hand, by deficiencies in the description of the degrees of freedom related to the core, and third, by the lack of sufficient experimental information. The aim of the present work is, while staying at the point of view of the experimentalists, to

apply a contemporary version of the particle plus rotor model in order to investigate the possibility of obtaining a reasonable description of the properties of  $^{195}\text{Pt}$  within such a simple approach. We concentrate exclusively on the negative-parity states for which a very rich set of experimental information is available and which dominate the spectrum at low excitation energy.

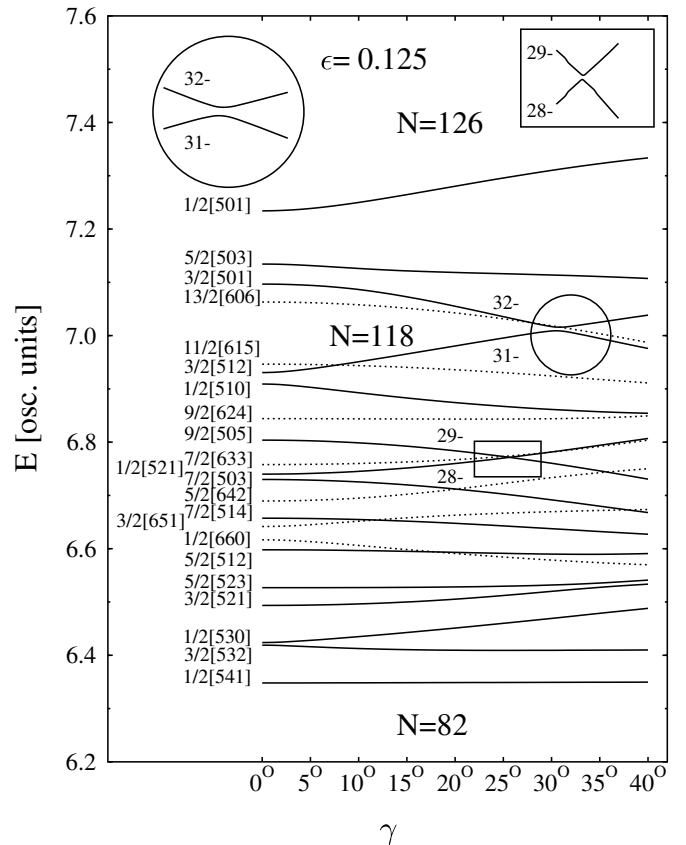
## 2 The model and the calculations

In the present work, we used the particle plus triaxial rotor model (PTRM) of ref. [9]. Referring the reader for more details to that paper, we describe here only the main features of the model. First, the single-particle energies and wave functions corresponding to the modified harmonic oscillator (MHO) potential are calculated for fixed quadrupole deformation ( $\epsilon, \gamma$ ). From the generated Nilsson states a set is selected which is used to construct the particle plus rotor strong-coupling basis states. Within this set of orbitals all single-particle matrix elements necessary for the particle plus rotor Hamiltonian and the calculation of transition strengths are computed. The residual pairing interaction is treated within the BCS approximation and a Fermi level  $\lambda$ , pairing gap  $\Delta$  and quasi-particle energies are derived. As a next step, the particle plus triaxial rotor Hamiltonian matrix is constructed and diagonalized in the one-quasi-particle strong-coupling basis for a selected range of the total spin  $I$ . Here, the core energy spectrum is taken into account by specifying

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the hydrodynamic moments of inertia of the triaxial rotor. Then, electromagnetic matrix elements, both diagonal and off-diagonal, are calculated with the wave functions obtained.

To perform the PTRM calculations, we used the computer codes GAMPN, ASYRMO, and PROBAMO presented in refs. [10,11]. Thereby, all 15 negative-parity single-neutron orbitals in the deformed  $N = 82$ –126 shell were taken into account and the standard set [12] of Nilsson parameters  $\kappa$  and  $\mu$  was employed. The attenuation  $\zeta$  of the Coriolis interaction was treated as an adjustable parameter. The moments of inertia, which depend on  $\epsilon$  and  $\gamma$  according to the irrotational formulae, were adjusted too by varying the energy  $E_{2_1^+}$  of the  $2_1^+$  level of the effective even-even core. With the alternative use of a variable moment of inertia, employing Harris parameters derived from the neighboring even-even nuclei, it was not possible to obtain a better description of the energies of the excited states. The aim of the calculations was to obtain a good overall description of the level energies and transition strengths with only four parameters. This was achieved with the optimal values of the deformation parameters  $\epsilon = 0.125$  and  $\gamma = 32^\circ$ , and the values  $E_{2_1^+} = 230$  keV and  $\zeta = 0.6$ . We note that the  $E_{2_1^+}$  value of the effective even-even core differs from the value characterizing the  $^{194}\text{Pt}$  core ( $E_{2_1^+}(^{194}\text{Pt}) = 328$  keV). A value of  $\Delta = 0.79$  MeV was obtained by the BCS calculation for the pairing gap. For the Fermi level, that calculation yielded  $\lambda_F = 52.71$  MeV (*i.e.*, 6.98 oscillator units). Values from 0.6 to 0.8 for the parameter  $\zeta$  are typically used to describe the attenuation of the Coriolis interaction (*cf.*, *e.g.*, ref. [13]). Our value of  $\zeta = 0.6$  slightly differs from the value 0.7 used in the previous rotor plus quasi-particle calculations of ref. [8]. A deviation from axial symmetry ( $\gamma \neq 0$ ) was found to be essential for the correct reproduction of the relative positions of the levels and especially of the  $I^\pi = 1/2^-$  ground state. The deformation parameter  $\epsilon$  was mainly fixed with regard to the reproduction of the strong  $E2$  transition from the  $I^\pi = 5/2^-$  level at 240 keV to the  $1/2^-$  ground state. While the dependence of the single-particle energies on the deformation parameter  $\epsilon$  ( $\beta$ ) is very well known, their  $\gamma$ -dependence is not so familiar. To illustrate it for the particular case discussed and give an idea on the active neutrons orbitals in  $^{195}\text{Pt}$ , we show in fig. 1 the energies of the single-particles states calculated at the optimal  $\epsilon$  value. The most interesting feature relevant to our PTRM calculations is the non-crossing or quasi-crossing of the orbitals with numbers 31 and 32 (the numbering is among the negative-parity orbitals). The odd neutron in  $^{195}\text{Pt}_{117}$  should lie on the orbital No. 31 ( $\pi = -$ ) for values of  $\gamma$  below  $30.7^\circ$  (where the quasi-crossing occurs) and on the continuation of orbital No. 32 ( $\pi = -$ ) above. The quasi-crossing of the two orbitals occurs very close to the optimal value of  $\gamma$  found in the PTRM calculations. As the calculations show and as it will be discussed below, the orbitals 31 and 32 dominate the structure of the low-lying states. For the calculation of the  $E2$  transition strengths the macroscopically determined quadrupole mo-

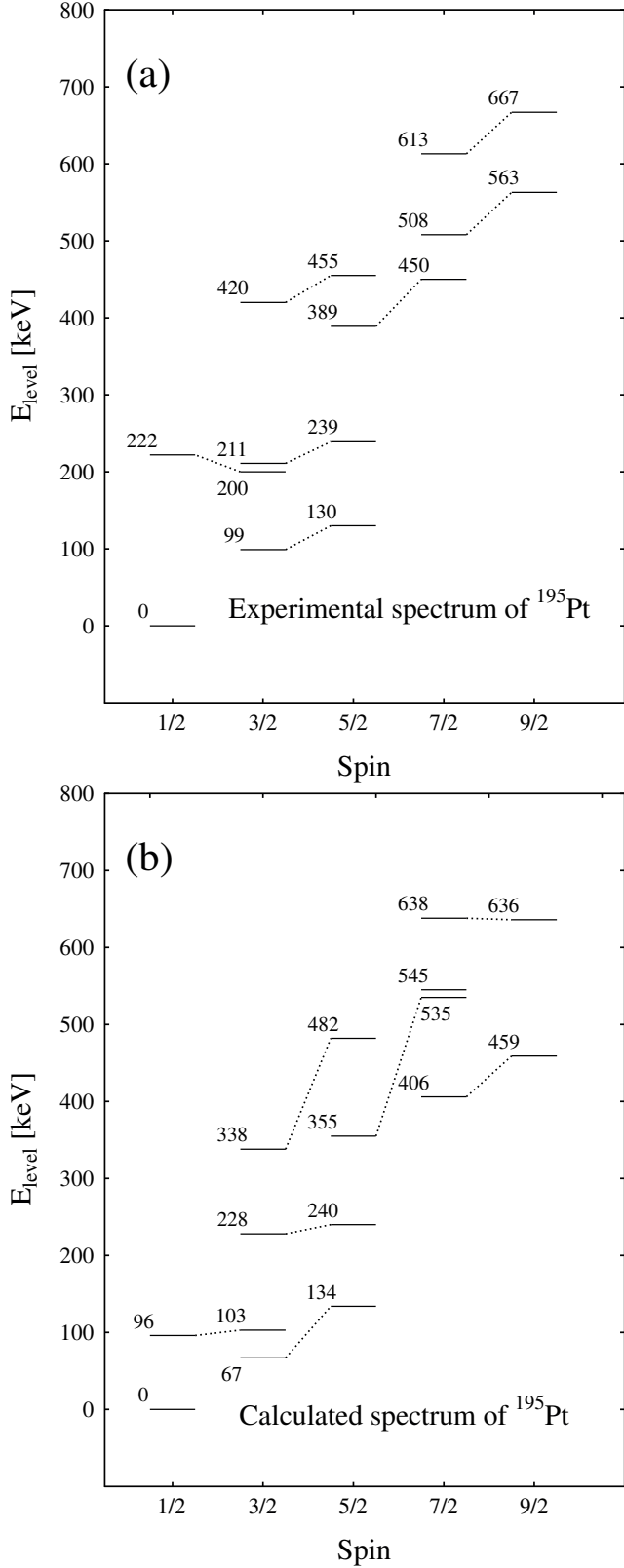


**Fig. 1.** Single-particles states in the  $N = 82$ –126 shell calculated for a fixed value of  $\epsilon$  and different values of  $\gamma$ . The negative-parity states are shown with solid lines while the positive-parity ones are shown with dotted lines. At  $\gamma = 0^\circ$ , the orbitals are labeled with the asymptotic Nilsson quantum numbers  $\Omega[Nn_zA]$ . The inserts emphasize the non-crossing of two pairs of negative-parity orbitals. The minimal distance between the orbitals  $31(\pi = -)$  and  $32(\pi = -)$  is 51 keV and that between the orbitals  $28(\pi = -)$  and  $29(\pi = -)$  is about 1.9 keV. In the figure, one oscillator unit is equal to about 7.56 MeV. For  $^{195}\text{Pt}$ , at  $\gamma = 32^\circ$  the Fermi level is positioned at 6.98 oscillator units. See also text.

ments of the core were used apart from the single-particle contributions. For the  $M1$  transition strengths and magnetic moments the neutron  $g_s$ -factor was reduced to 0.7 of its free value and the core factor  $g_R$  was fixed to  $Z/A$ .

### 3 Results and discussion

The experimental spectrum of the negative-parity excited states of  $^{195}\text{Pt}$  which lie below 700 keV is shown in fig. 2a. Only two levels, the  $I^\pi = 3/2^-$  at 525 keV and the  $I^\pi = 5/2^-$  at 544 keV, are not included (*cf.* below). The striking feature of the data is that all levels displayed can be grouped in doublets consisting of levels which are energetically close to each other and whose spins differ by  $1\hbar$ . This fact is closely related to the supersymmetry (SUSY) interpretation (*cf.*, *e.g.*, ref. [2] and references therein) of the level structures in  $^{194}\text{Pt}$  and  $^{195}\text{Pt}$ . When group-



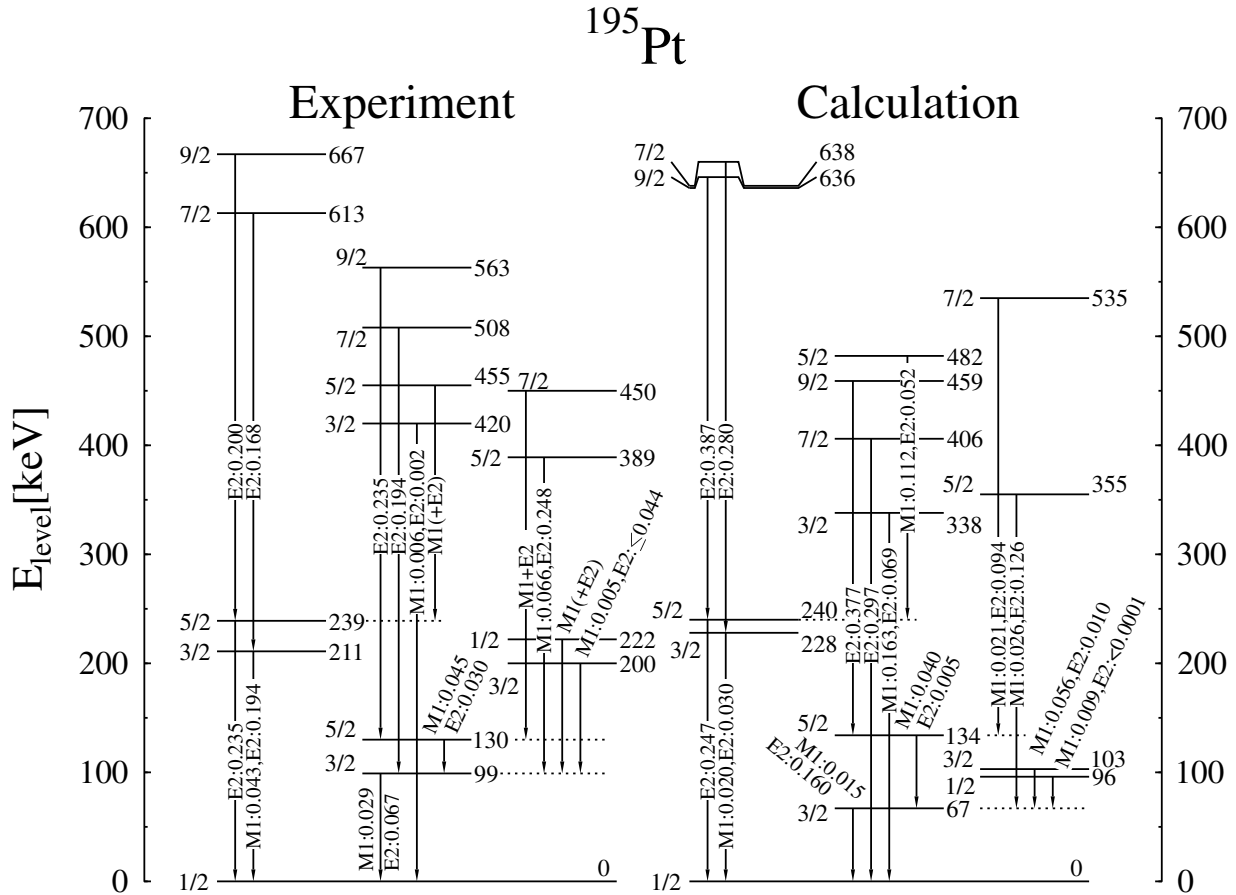
**Fig. 2.** a) Experimentally observed excited states with negative parity in  $^{195}\text{Pt}$  below 700 keV displayed *versus* their spin. Levels forming doublets with spin difference  $\Delta I = 1\hbar$  are connected with a dotted line. The data are taken from refs. [14, 15, 2]. b) Spectrum of the negative-parity excited states in  $^{195}\text{Pt}$  calculated within the framework of PTRM. See also text.

**Table 1.** Experimental reduced electromagnetic transition probabilities  $B(\sigma L)$  compared to the results of the calculation. They are shown in units of  $e^2 b^L$  for electric transitions and of  $\mu_N^2 b^{L-1}$  for magnetic ones. The data are taken from refs. [14, 15]. The energies of the initial and final levels as well their spin/parity are also shown. The multipolarity  $\sigma L$  of the depopulating  $\gamma$ -ray transitions is displayed in column 6.

$E_i$ (keV)	$I_i^\pi$	$E_\gamma$ (keV)	$E_f$ (keV)	$I_f^\pi$	$\sigma L$	$B(\sigma L)_{exp}$	$B(\sigma L)_{th}$
98.9	3/2	98.9	0	1/2	M1	0.029(4)	0.015
					E2	0.067(9)	0.160
129.8	5/2	30.9	98.9	3/2	M1	0.045(3)	0.040
					E2	0.030(13)	0.005
199.5	3/2	129.8	0	1/2	E2	0.064(9)	0.014
		199.5	0	1/2	M1	0.005(2)	0.056
					E2	$\leq 0.044$	0.010
					E2	0.029(9)	0.018
211.4	3/2	211.4	0	1/2	E2	0.194(47)	0.030
239.3	5/2	140.9	98.9	3/2	E2	0.054(27)	0.044
389.2	5/2	150.1	239.3	5/2	M1	0.054(27)	0.017
		259.4	129.8	5/2	M1	0.045(21)	0.014
					E2	0.0002(1)	0.003
					E2	0.248(134)	0.126
419.7	3/2	197.5	222.2	1/2	M1	$> 0.001$	0.035
					E2	$< 0.235$	0.002
					M1	0.0023(5)	0.124
					M1	0.006(1)	0.163
508.1	7/2	119.1	389.2	5/2	M1	$< 0.25$	0.018
					E2	$< 26$	0.128
					E2	0.047(20)	0.0003
					E2	0.194(67)	0.297
562.8	9/2	432.9	129.8	5/2	E2		
612.7	7/2	373.4	239.3	5/2	M1	$< 0.138$	0.041
					E2	$< 1.407$	0.011
					E2	0.168(107)	0.280
667.1	9/2	428	239.3	5/2	E2	0.12(4)	$< 0.0001$
		537	129.8	5/2	E2		

ing the experimental levels in doublets we follow fig. 4 in ref. [2]. The spectrum calculated within the framework of the PTRM is shown in fig. 2b while the theoretical electromagnetic properties of the levels were compared to the experimental ones in tables 1 and 2.

The closely lying  $\Delta I = 1$  calculated levels can also be grouped in doublets to check if the experimental pattern is reproduced. This grouping, illustrated by the dotted lines in fig. 2b, was made after the assignment of every calculated level to an experimental one. For this purpose, the electromagnetic decay properties of the levels were considered. Namely, strong ( $> 0.1 e^2 b^2$ , *i.e.*  $> 15$  W.u.)



**Fig. 3.** Experimental partial level scheme of  $^{195}\text{Pt}$  from refs. [14,15,2] compared to the PTRM calculations. The  $B(E2)$  and  $B(M1)$  transition strengths are given in  $e^2b^2$  and  $\mu_N^2$ , respectively, for the strongest transition depopulating a given level. See also text.

**Table 2.** Experimental and calculated magnetic moments of levels in  $^{195}\text{Pt}$  in units of  $\mu_N$ . The level energies are displayed in the first row. The data are taken from from refs. [14,15].

$E$ (keV)	0	99	130	211	239	389	508	563	613	667
Spin ( $\hbar$ )	1/2	3/2	5/2	3/2	5/2	5/2	7/2	9/2	7/2	9/2
$\mu_{exp}$	0.60952(6)	-0.62(6)	0.90(6)	0.156(32)	0.523(50)	0.39(10)	0.55(8)	1.55(12)	1.44(42)	1.52(16)
$\mu_{th}$	0.55	0.23	1.01	0.40	0.98	1.00	0.95	1.84	1.32	1.60

$B(E2)$  transition strengths, when observed experimentally, were also required between the corresponding calculated levels. This condition fixed the (1/2, 3/2), the two lowest (3/2, 5/2) and the two (7/2, 9/2) doublets. After this step, the calculated levels with  $I^\pi = 3/2^-$  at 338 keV,  $I^\pi = 5/2^-$  at 482 keV,  $I^\pi = 7/2^-$  at 535 keV, and  $I^\pi = 7/2^-$  at 545 keV remained to be classified. The assignment of the theoretical  $I = 3/2$  and  $I = 5/2$  levels is natural, since these are the only low-lying available states with convenient spins. (It should be mentioned that the PTRM calculation predicts more levels at higher excitation energy which are not shown in fig. 2b). The case of the  $I^\pi = 7/2^-$  levels is more complicated since one of them appears as an extra state in the energy range considered. We prefer the assignment of the level at 535 keV to

the experimental level at 450 keV because of the predicted mixed character ( $M1 + E2$ ) of the strongest experimentally established transition depopulating this state. Concerning the positive-parity states, the lowest predicted one has  $I^\pi = 13/2^+$  and lies at 154 keV. Experimentally, the lowest positive-parity state has the same spin and lies at 259 keV. Five positive-parity states are predicted in the excitation energy range up to 800 keV in a reasonable agreement with the experiment.

The strongest transitions depopulating the experimental levels are compared to the theoretical predictions in fig. 3 which illustrates also the degree of agreement obtained for the level energies, in addition to figs. 2a and b. The overall description of the level scheme is acceptable, but not all features are completely reproduced. Drawbacks

of the calculations are the precision of the description of the energy splitting of the doublets and the positions of some levels as well as the degree of reproduction of a number of  $B(E2)$  and  $B(M1)$  transition strengths.

The agreement for the strong  $E2$  transitions is within a factor of two, with the exception of the transition from the experimental  $I^\pi = 3/2^-$  level at 211 keV to the ground state. Presumably, the precise structure of this level is difficult to attain since it is lying experimentally only 11 keV apart from another  $I^\pi = 3/2^-$  level and the mixing between them is probably not well described by the PTRM calculations. The description of the weak transition strengths in about half of the cases is also reasonable (within a factor of 3) but there are cases where the discrepancy exceeds one order of magnitude. The latter can be attributed to the sensitivity of the transition matrix elements with respect to the wave functions of the initial and final states. The PTRM calculations predict that the negative-parity orbitals with numbers 31 and 32, which lie close to the Fermi level, completely dominate the structure of the low-lying states. Only at higher excitation energies complementary orbitals start to appear as main components of the wave functions. Such domination of additional orbitals in possible calculated doublet partners was the reason to exclude from the comparison the above-mentioned levels with  $I^\pi = 3/2^-$  at 525 keV and  $I^\pi = 5/2^-$  at 544 keV. The importance of the negative-parity orbitals 31 and 32 for the structure of the low-lying states can be expected from fig. 1, however, the calculation of the precise amplitudes may represent an overwhelming task for the PTRM due to the simplifications of the model. Such simplifications are for instance the assumption of a rigid core, the BCS treatment of the pairing with only approximate inclusion of the blocking effect, impossibility to calculate self-consistently the quadrupole deformation for each excited state, the employed core-particle coupling and so on. One may hope that a more sophisticated model, which takes better into account the degrees of freedom of the core and fully incorporates the feature of closeness in energy and non-crossing of the orbitals with numbers 31 and 32, will be able to give a better description of the data. This comment applies also for the magnetic moments shown in table 2. Their overall description is satisfactory, especially at higher spins, with the exception of the large discrepancy for the  $3/2^-$  level at 99 keV, where the experimental magnetic moment is negative in contradiction to the calculation. The reason for this disagreement is not clear for us. It should be mentioned that the agreement between theory and experiment in this case is better within the SUSY approach.

Coming back to the closeness in energy and non-crossing of the orbitals with numbers 31 and 32 illustrated in fig. 1, it is very interesting that it occurs in the vicinity of  $\gamma = 30^\circ$ , a value of the nuclear-shape asymmetry parameter associated with the even-even cores of the mass-region. We remind here that the nuclei  $^{194,196}\text{Pt}$  are considered as ones of the best examples for a realization of the  $O(6)$ -symmetry limit of the IBA [16]. Although it is not clear if the triaxiality is dynamic, an effective expression

of  $\gamma$ -softness, or static, as in the model employed, it is of importance for the description of the low-lying states. For instance, it is well known that the IBA predicts an effective triaxiality of  $30^\circ$  for an  $O(6)$  nucleus. On the other hand, the observation of the non-crossing may be informative for the existence of good quantum numbers, aside from parity, characterizing the single-particle motion in triaxial nuclei. This point is beyond the scope of the present work, but surely deserves dedicated theoretical efforts in order to be elucidated. At this stage, we remark too the analogy of the closeness in energy of the two active negative-parity orbitals, which is reflected also in their very similar quasi-particle energies, and the doublet structure of the experimental spectrum. The orbital 31 is associated with the  $3/2[512]$  Nilsson state at  $\gamma = 0^\circ$ . Its partner, orbital 32, is associated with the  $3/2[501]$  Nilsson state. The main components of the wave functions of the two orbitals originate from the  $f_{5/2}$ ,  $p_{3/2}$  and  $p_{1/2}$  spherical subshells. These subshells form the fermion space involved in the SUSY interpretation of the level scheme of  $^{195}\text{Pt}$ .

Concerning the comparison with previous particle-rotor calculations, to our knowledge in the literature for  $^{195}\text{Pt}$  there is not a more detailed description of the level energies below 700 keV and especially of the corresponding transition strengths than the one given by the present work. Historically, Hecht and Satchler [7] were the first to apply an asymmetric rotor plus particle model in an attempt to describe the properties of  $^{195}\text{Pt}$ . Their conclusion was that the observed level energies can be reproduced by the model but the measured transition strengths are not in agreement with the theoretical predictions. The main drawback was the impossibility to describe the relatively strong  $B(E2)$  values of the transitions from the first two  $(3/2, 5/2)$  doublets to the ground state. However, Hecht and Satchler employed a rather simple version of the model where only one-particle orbital is coupled to the core. They pointed out that the assumption of two different particle excitations responsible for the low-lying states may be reasonable and could lead to a better description of the transition strengths. It should also be noticed that at the time of their work the experimental information on  $^{195}\text{Pt}$  was quite limited. All this discouraged further applications of the particle plus triaxial rotor model to that nucleus. Instead, other theoretical approaches have been tried. We consider briefly below only those where attempts were made for a large-scale reproduction of the level scheme of  $^{195}\text{Pt}$ . Using  $(d, p)$  and  $(d, t)$  reactions, Yamazaki and Sheline have determined spectroscopic factors and tried to describe them and the level energies using three different models [8]. Especially, they performed calculations in the framework of the Nilsson model [17] including Coriolis and pairing interaction and the Faessler and Greiner model (ref. [18] and references therein), and also tried to employ an extension of the Davydov and Filippov [19] model for odd nuclei. The comparison with the data favors the rotation-vibration model of Faessler and Greiner, which assumes a  $\gamma$ -soft core over the model of Nilsson with assumed oblate nuclear shape. Due to the limitation of the numerical procedures at that time, the

extension of the model of Davydov and Fillipov with several orbitals coupled to the core and pairing did not give satisfactory results, but was not totally rejected by the authors. Yamazaki and Sheline did not calculate electromagnetic transition strengths in their work [8]. Concerning the description of the level energies, their results are comparable to those from the present work. A later work [20] on levels in  $^{195}\text{Pt}$  excited via neutron pickup confirmed the experimental results of ref. [8] but pointed out some disagreement between them and the theory presented in [8]. The conclusion was rather that there is not a single model which can completely describe the data. This conclusion became more and more weakened with time since the introduction by F. Iachello [3], in the early eighties, of the ideas of supersymmetry (SUSY) [3] for the description of the nuclear excited states. Without going into details, we refer the reader to some of the works where important steps were made in the understanding of the structure of  $^{195}\text{Pt}$  [2, 21–29]. In simple terms, the even-even core, *i.e.* the boson system, and the odd fermion are treated simultaneously using the group  $U(6/12)$ . Explicitly, for  $^{195}\text{Pt}$ , the  $O(6)$  core and the  $p_{1/2}$ ,  $p_{3/2}$  and  $f_{5/2}$  neutron orbitals constitute the valence space of the model which provides a simultaneous description of  $^{194}\text{Pt}$  and  $^{195}\text{Pt}$ . The degree of agreement for the level energies (*e.g.*, refs. [2, 29]) is good and comparable to or better than that of the PTRM calculations from the present work. Concerning the transition strengths,  $B(E2)$  and  $B(M1)$  values as well as  $g$ -factors have been calculated and extensively discussed [25–27]. For them, the degree of agreement is also comparable and generally better than the one found for our PTRM calculations. It should be mentioned here that the SUSY approach implies the use of nine parameters (five for the level energies and four for the electromagnetic  $E2$  and  $M1$  operators) whereas we employed for the description of  $^{195}\text{Pt}$  within the particle plus triaxial rotor model only four parameters. However, a simple comparison of the number of parameters seems to be not possible since the latter model involves some fixed parameters like the set of  $\mu$  and  $\kappa$  values for the different  $N$ -shells. Although their values are considered to be well established in the mass region considered, they could be in principle varied too.

## 4 Summary and conclusions

The particle plus triaxial rotor model has been applied for  $^{195}\text{Pt}$ . An overall reasonable description of the level energies below 700 keV and of the corresponding  $B(E2)$  and  $B(M1)$  transition strengths was obtained with only four parameters:  $\epsilon$ ,  $\gamma$ ,  $E_{2^+_1}$  and  $\zeta$ . The importance of the closeness in energy of the two active negative-parity neutron orbitals, which are positioned near to the Fermi surface at  $\gamma \approx 30^\circ$ , for the understanding of the structure of the low-lying levels, is pointed out. Further theoretical efforts, especially with respect to accounting for the properties of the core and the particle-core coupling, are required for a more successful description of  $^{195}\text{Pt}$  in a particle plus rotor picture.

We are grateful to I. Ragnarsson and P. Semmes for helpful discussions and providing us with the rotor plus quasi-particle codes. We further thank R.F. Casten, A. Dewald, G. Graw, R. Hertenberger, A. Gelberg, R.V. Jolos, T. Otsuka and C. Scholl. The authors are indebted to the DFG for the financial support under Contracts Jo 391/2-3 and Br 799/12-1. P.P. appreciates the kind hospitality of the University of Cologne.

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